For Supervisor's use only

3

90638

NEW ZEALAND QUALIFICATIONS AUTHORITY
MANA TOHU MĀTAURANGA O AOTEAROA

National Certificate of Educational Achievement
TAUMATA MĀTAURANGA Ā-MOTU KUA TAEA

Level 3 Calculus, 2004

90638 Manipulate real and complex numbers, and solve equations

Credits: Five
9.30 am Tuesday 23 November 2004

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

Make sure you have a copy of the Formulae and Tables booklet L3–CALCF.

You should answer ALL the questions in this booklet.

Show ALL working for ALL questions.

If you need more space for any answer, use the pages provided at the back of this booklet and clearly number the question.

Check that this booklet has pages 2–9 in the correct order and that none of these pages is blank.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

Achievement Criteria	*For Assessor's use only*	
Achievement	**Achievement with Merit**	**Achievement with Excellence**
Manipulate real and complex numbers, and solve equations. ☐	Solve more complicated equations. ☐	Solve problem(s) involving real or complex numbers. ☐
Overall Level of Performance ☐		

You are advised to spend 40 minutes answering the questions in this booklet.

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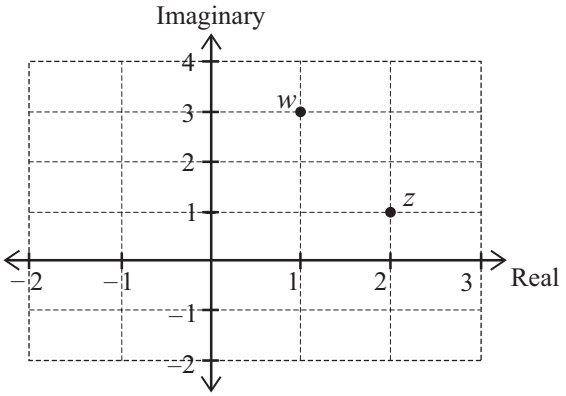
Show **ALL** working.

QUESTION ONE

(a)



(i) Write wz in rectangular form, $a + b\text{i}$.

(ii) Write $\overline{z}$, the conjugate of z, in polar form, $r\ \text{cis}\,\theta$.

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(b) If $u = 3\,\text{cis}\,1.2$ and $v = 5\,\text{cis}\,0.4$, then write $\frac{u}{v}$ in polar form, $r\,\text{cis}\,\theta$.

(c) Write $\frac{6+\sqrt{2}}{2-\sqrt{2}}$ in $a+b\sqrt{2}$ form, where a and b are rational numbers.

(d) Write $(2\,\text{cis}\,\frac{\pi}{3})^{12}$ in rectangular form, $a + b\text{i}$.

QUESTION TWO

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(a) Solve $3^{2x-1} = 4$, giving your answer to at least 3 significant figures.

(b) Solve $(x+3)^2 = 2x+2$.

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(c) Solve $\log_3 x = 2 + \log_3(x-2)$.

QUESTION THREE

Solve $x+1=\sqrt{6-2x}$.

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QUESTION FOUR

Find the square roots of the complex number $5-12\mathrm{i}$.

QUESTION FIVE

One root of the equation $z^3-6z^2+13z+k=0$, where k is real, is $z=2+\mathrm{i}$.

Find the value of k and the other two roots.

QUESTION SIX

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w is one of the complex cube roots of 1.

(a) Show that $w^2 + w + 1 = 0$.

(b) Prove that $\dfrac{1}{w^2 + w^4} = -1$.

(c) Given that the conjugate of w is equal to w^2, find the conjugate of $1 + w$ in terms of w.

Extra paper for continuation of answers if required.
Clearly number the question.

Assessor's use only

Question number

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Clearly number the question.

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